Assignment- Differential Calculus & Equations

Last date of Submission: 26/02/2020. **MM: 20**

Note: Do any five questions.

1) **Topic: Differential Calculus.**

Q1. If you had a device that could record the temperature of a room continuously over a 24-hour period, would you expect the graph of temperature versus time to be a continuous (unbroken) curve? Explain your reasoning.

Q2. Sketch the graph of a continuous *f* with stated properties:

(a) *f* is concave upward on the interval $(-\infty,+\infty)$ and has no relative extrema.
(b) *f* has exactly two relative extrema on $(-\infty,+\infty)$ and $f(x) \to 0$ as $x \to -\infty$ and as $x \to +\infty$.

Q3. Give an example of a function *f* that is defined on a closed interval, and whose values at the end points have opposite signs, but for which the equation $f(x) = 0$ has no solution in the interval.

2) **Topic: Linear differential equation of first order:**

Q4. A point P is dragged along the xy-plane by a string PT of length a. If T starts at the origin and moves along the positive y- axis, and if P starts at (a, 0), what is the path of P?

Q5. A rabbit starts at the origin and runs up the y-axis with speed a. At the same time a dog, running with speed b, starts at the point (c, 0) and pursues the rabbit. What is the path of the dog?

3) **Topic: Linear differential equation of second order.**

Q6. Consider a cart of mass M attached to a nearby wall by means of a spring. The spring exerts force when the cart is at equilibrium position x = 0. If the cart is displaced by a distance x, then the spring exerts a restoring force $F_s = -kx$,(Simple harmonic motion) where k is a positive constant whose magnitude is a measure of the stiffness of the spring. Find the position of cart at any time t, when released without any initial velocity at time t = 0. Also find the amplitude and frequency.[**Hint**: Form the differential equation by using newton's second law of motion, F_s = Mass $\times$ acceleration].

Books suggested:

1) "Calculus" by Howard Anton 10th edition.
2) "Calculus" Volume 1 by T M Apostol.
3) Differential equation with its application by George F. Simmons.
4) Differential equation by S L Ross.

Note: Student will have to give the detail of the references for each question.

Submitted to
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